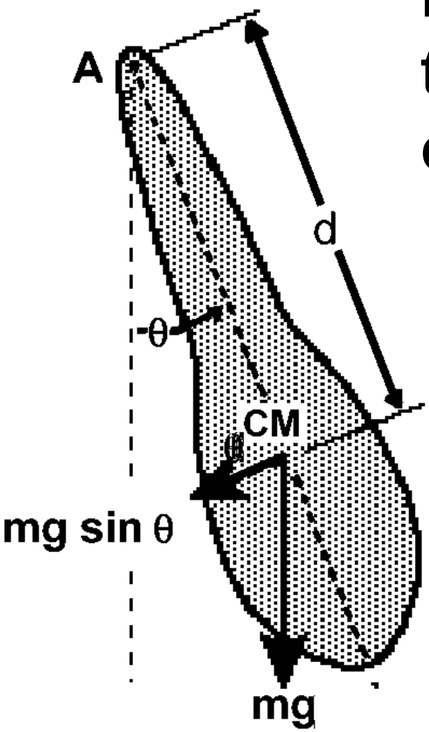
Physical pendulum

- **An object swinging back and forth also undergoes simple harmonic motion**
- **Such an object is called a physical pendulum.**

Simple pendulum

- **A simple pendulum is an idealized example of a physical pendulum, consisting of a massless rod with a point mass at its end.**

Physical pendulum



The object has mass m and moment of inertia *I* about *A*. The distance from *A* to the center of mass *CM* is *d*. We can see the object when the displacement is θ,

The torque is $\Gamma = rF\sin\theta$

- r- is the length of the line from A to CM,
- F sin θ - is the component of the force perpendicular to this line.
- r = d and F = mg.

$$\Gamma = -mgd\sin\theta$$

Physical pendulum

Using the angular form of Newton's second law, $\Gamma = I\alpha$

α - angular acceleration

we have $-mgd\sin\theta = I\alpha$ or $\alpha = -\frac{mgd}{I}\sin\theta$

At small angles sin θ and θ are almost equal.

Thus for small angles we have $\alpha = -\frac{mgd}{I}\theta$

This has the correct form for harmonic angular motion,

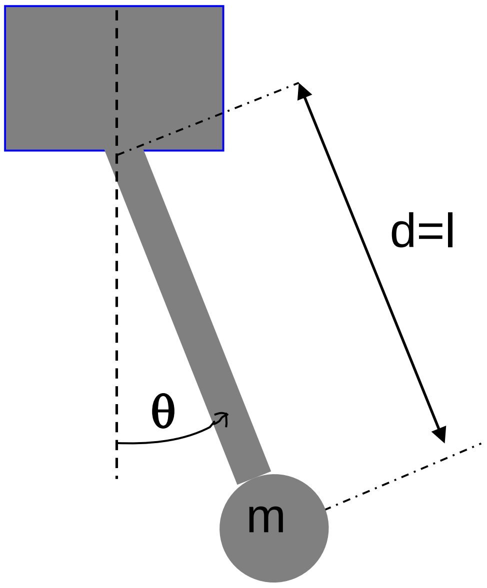
$\alpha = -(2\pi f)^2\theta$ **if we identify the characteristic frequency as**

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}}$$

or

$$T = 2\pi\sqrt{\frac{I}{mgd}}$$

Simple pendulum



The simple pendulum consists of a point mass *m* at one end of a massless rod of length *l* pivoted about the other end.

Its moment of inertia is $$I = ml^2$$

and d = l.

Using the results for the physical pendulum, the frequency of the simple pendulum is

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{mgl}{ml^2}} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$

and the period of motion is then $$T = 2\pi\sqrt{\frac{l}{g}}$$

Damped oscillations

The dissipative force F_d is linearly proportional to v

$$F = -\gamma v$$

γ **- is the damping constant**

Forced oscillations

- **Unless energy is supplied, the amplitude of an oscillator usually decreases in time because of frictional effects.**
- **When energy is fed into a vibrating system, it is said to be undergoing forced oscillations**

Resonance

- Energy is most effectively supplied to an oscillator when the external force acts at the correct frequency, which is usually closed to the frequency of the oscillator with no external force.
- This phenomenon is called resonance.
- The optimum frequency if referred to as the resonant frequency.